

Topic 6: Investigating Volume

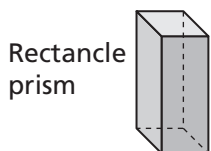
for use before **Looking for Pythagoras**

Investigation 3

Volume is the amount of space enclosed in a solid. It is expressed in cubic units. The volume of the Empire State Building in New York City is about 37 million cubic feet. The volume of a raisin is about $\frac{1}{8}$ cubic inch.

Problem 6.1

- a. The volume of a prism or a pyramid with a square base is determined by the height of the solid and the area of the base.



$$\begin{aligned}\text{Volume} &= \text{Base} \times \text{height} \\ \text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= lwh\end{aligned}$$

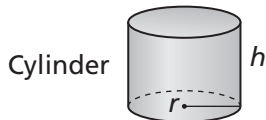


$$\begin{aligned}\text{Volume} &= \frac{1}{3} \times \text{Base} \times \text{height} \\ &= \frac{1}{3}s^2h\end{aligned}$$

- A. What is the volume of a rectangular prism with a length of 8 cm, a width of 4 cm, and a height of 10 cm?
- B. What is the volume of a square pyramid with a height of 3 feet and a base with sides of 1 foot?
- C. Give two sets of possible dimensions for a rectangular prism with a volume of 100 in^3 .
- D. How does the volume of a rectangular prism change if you lay it on its side before taking measurements? Explain.

Problem 6.2

The volume of a cone or a cylinder is determined by the height and the area of the base. The exact volume of a cone or cylinder includes the value π .



$$\begin{aligned}\text{Volume} &= \text{Base} \times \text{height} \\ &= \pi r^2 h\end{aligned}$$



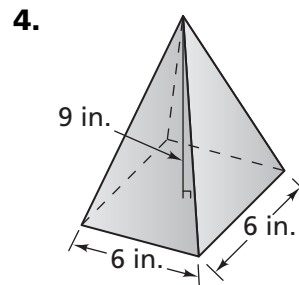
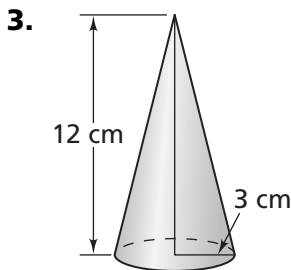
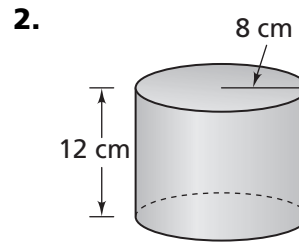
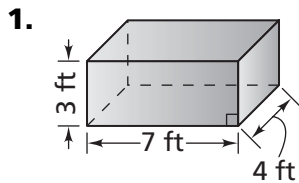
$$\begin{aligned}\text{Volume} &= \frac{1}{3} \times \text{Base} \times \text{height} \\ &= \frac{1}{3} \pi r^2 h\end{aligned}$$

- A. 1. What is the exact volume of a cylinder with radius 5 m and height 3 m?
2. What is the exact volume of a cone with radius 7 in. and height 12 in.?

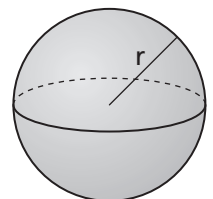
- B.** If a cone and a cylinder have the same radius and the same height, how many times greater is the volume of the cylinder than the cone?
- C.** If the volume and height of a cone and a cylinder are the same, which one has the larger base?
- D.** If the radius of a cone doubles and the height remains the same, what happens to the volume?

Exercises

Find the exact volume of each solid.



- 5.** How are the formulas for the volume of prisms and cylinders the same?
- 6.** How are the formulas for the volume of cones and pyramids the same?
- 7.** A watering can does not fit under a faucet, so Trang is using a paper cone to fill it with water. The cone has a radius of 1 inch and a height of 4 inches. The can has a radius of 2 inches and a height of 8 inches. How many cone-fuls of water will it take to fill the can?
- 8.** The Great Pyramid of Khufu in Egypt has a square base with sides of about 230 meters and a height of about 146 meters. What is the approximate volume of the pyramid?
- 9.** The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.
- What is the volume of a baseball with a diameter of 2.8 inches?
 - About many times more volume does a basketball with a radius of 4.78 inches have than a baseball?
- 10.** What is the exact volume of the largest cone that can fit into a cube with sides of 10 inches?



Topic 6: Investigating Volume

PACING 1 day

Mathematical Goals

- Determine the volume of three-dimensional shapes.
- Investigate the relationship of the volume of three-dimensional figures with the same base area and height.

Guided Instruction

Use unit cubes in an exercise to give the students a hands-on experience with cubic measure. Show that the cubes can be rearranged into numerous configurations without the volume changing.

The solid shapes used in this lesson are all right shapes: the bases of the rectangular prisms and cylinders are aligned vertically; the apexes of the pyramids and cones are directly above the center of the base. Even if these shapes leaned to one side, the same formulas for volume would apply.

After Problem 6.1

- *Why doesn't it matter which edges you call length, width, and height when you find the volume of a rectangular prism?* (According to the Commutative Property of Multiplication it doesn't matter in what order you do the multiplication.)
- *How do you define a rectangular prism?* (a solid figure with six faces that are rectangles)
- *What do you call a rectangular prism whose sides are all squares?* (cube)
- *What is the definition of a cubic inch?* (The volume of a cube with sides of 1 inch \times 1 inch.)

You will find additional work on volume in the grade 7 unit *Filling and Wrapping*.

Vocabulary

- volume

Materials

- Labsheet 6ACE Exercises 1–4

Assignment Guide for Topic 6

Core 1–9

Advanced 10

Answers to Topic 6

Problem 6.1

- A. 1. 320 cm^3
- B. 1 ft^3
- C. Answers may vary. Sample: $5 \text{ in.} \times 5 \text{ in.} \times 4 \text{ in.}$; $10 \text{ in.} \times 5 \text{ in.} \times 2 \text{ in.}$
- D. The volume does not change if you place a rectangular prism on its side before measuring.

Problem 6.2

- A. 1. $75\pi \text{ m}^3$
2. $196\pi \text{ in.}^3$
- B. 3 times
- C. the cone
- D. Volume increases by a factor of 4.

Exercises

- 1. 84 ft^3
- 2. $768\pi \text{ cm}^3$
- 3. $36\pi \text{ cm}^3$
- 4. 108 in.^3
- 5. They both multiply the area of a base times the height.
- 6. They both multiply $\frac{1}{3}$ the area of a base times the height.
- 7. 24 cone-fuls
- 8. 2,574,467 m^3
- 9. a. 11.5 in.^3
b. 40 times more volume
- 10. $\frac{250\pi}{3} \text{ in.}^3$

Labsheet 6ACE Exercises 1-4

Topic 6

